

Inference at \*  
of proof for Lemma wellfounded\_functionality\_wrt\_implies:

$\vdash \forall T_1, T_2: \text{Type}, r_1: (T_1 \rightarrow T_1 \rightarrow \mathbb{P}), r_2: (T_2 \rightarrow T_2 \rightarrow \mathbb{P}).$   
 $(T_1 = T_2)$   
 $\Rightarrow (\forall x, y: T_1. \{r_1(x, y) \Leftarrow r_2(x, y)\})$   
 $\Rightarrow \{\text{WellFnd}\{i\}(T_1; x, y. r_1(x, y)) \Rightarrow \text{WellFnd}\{i\}(T_2; x, y. r_2(x, y))\}$   
by (((Unfolds “wellfounded guard“ 0)  
CollapseTHEN (UnivCD)).)  
CollapseTHEN ( (Auto\_aux (first\_nat 1:n) ((first\_nat 1:n), (first\_nat 3:n)) (first\_tok :t) inil\_term))) .

1:

1.  $T_1 : \text{Type}$
  2.  $T_2 : \text{Type}$
  3.  $r_1 : T_1 \rightarrow T_1 \rightarrow \mathbb{P}$
  4.  $r_2 : T_2 \rightarrow T_2 \rightarrow \mathbb{P}$
  5.  $T_1 = T_2$
  6.  $\forall x, y: T_1. r_1(x, y) \Leftarrow r_2(x, y)$
  7.  $\forall P: (T_1 \rightarrow \mathbb{P}). (\forall j: T_1. (\forall k: T_1. r_1(k, j) \Rightarrow P(k)) \Rightarrow P(j)) \Rightarrow \{\forall n: T_1. P(n)\}$
  8.  $P : T_2 \rightarrow \mathbb{P}$
  9.  $\forall j: T_2. (\forall k: T_2. r_2(k, j) \Rightarrow P(k)) \Rightarrow P(j)$
- $\vdash \forall n: T_2. P(n)$